

# NEUTRINO OSCILLATIONS, AND THE ORIGIN OF PULSAR VELOCITIES AND DARK MATTER

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Neutrino oscillations in a core-collapse supernova may be responsible for the observed rapid motions of pulsars. Three-dimensional numerical calculations show that, in the absence of neutrino oscillations, the recoil velocities of neutron stars should not exceed 200 km/s, while there exists a substantial population of pulsars that move faster than 1000 km/s. A small asymmetry in the neutrino emission may be the solution of this long-standing puzzle. Such an asymmetry could arise from neutrino oscillations, but, given the present bounds on the neutrino masses, the pulsar kicks require a sterile neutrino with a 1–20 keV mass and a small mixing with active neutrinos. The same particle can be the cosmological dark matter. Its existence can be confirmed by X-ray telescopes if they detect X-ray photons from the decays of the relic sterile neutrinos. One can also verify the neutrino kick mechanism by observing gravity waves from a nearby supernova.

## 1. Introduction

Neutrino oscillations in a core-collapse supernova have been the subject of intense studies. It has been pointed out, in particular, that neutrino oscillations may be the cause of otherwise unexplained large pulsar velocities<sup>1,2</sup>. This explanation requires the existence a sterile neutrino<sup>2,3</sup> with mass and mixing that are just right for the same particle to be the cosmological dark matter.

Pulsar velocities range from 100 to 1600 km/s<sup>4,5</sup>. Their distribution is non-gaussian, with about 15% of all pulsars having speeds over (1000 km/s)<sup>5</sup>. Pulsars are born in supernova explosions, so it would be natural to look for an explanation in the dynamics of the supernova<sup>6</sup>. However, state-of-the-art 3-dimensional numerical calculations<sup>7</sup> show that even

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the most extreme asymmetric explosions do not produce pulsar velocities greater than 200 km/s. Earlier 2-dimensional calculations<sup>8</sup> claimed a somewhat higher maximal pulsar velocity, up to 500 km/s. Of course, even that was way too small to explain the large population of pulsars whose speeds exceed 1000 km/s. Recent three-dimensional calculations by Fryer<sup>7</sup> show an even stronger discrepancy.

## 2. Why a sterile neutrino can give the pulsar a kick

Given the absence of a “standard” explanation, one is compelled to consider alternatives, possibly involving new physics. One of the reasons why the standard explanation fails is because most of the energy is carried away by neutrinos, which escape isotropically. The remaining momentum must be distributed with a substantial asymmetry to account for the large pulsar kick. In contrast, only a few per cent anisotropy in the distribution of neutrinos, would give the pulsar a kick of required magnitude.

Neutrinos are always *produced* with an asymmetry, but they *escape* isotropically. The asymmetry in production comes from the asymmetry in the basic weak interactions in the presence of a strong magnetic field. Indeed, if the electrons and other fermions are polarized by the magnetic field, the cross section of the urca processes, such as  $n + e^+ \rightleftharpoons p + \bar{\nu}_e$  and  $p + e^- \rightleftharpoons n + \nu_e$ , depends on the orientation of the neutrino momentum. Depending on the fraction of the electrons in the lowest Landau level, this asymmetry can be as large as 30%, much more than one needs to explain the pulsar kicks<sup>9</sup>. However, this asymmetry is completely washed out by scattering of neutrinos on their way out of the star<sup>14</sup>.

If, however, the same interactions produced a particle which had *even weaker* interactions with nuclear matter than neutrinos, such a particle could escape the star with an asymmetry equal its production asymmetry.

It is intriguing that the same particle can be the dark matter.

The simplest realization of this scenario is a model that adds only one singlet fermion to the Standard Model. The  $SU(2) \times U(1)$  singlet, a sterile neutrino, mixes with the usual neutrinos, for example, with the electron neutrino.

For a sufficiently small mixing angle between  $\nu_e$  and  $\nu_s$ , only one of the two mass eigenstates,  $\nu_1$ , is trapped. The orthogonal state,

$$|\nu_2\rangle = \cos\theta_m |\nu_s\rangle + \sin\theta_m |\nu_e\rangle, \quad (1)$$

escapes from the star freely. This state is produced in the same basic urca reactions ( $\nu_e + n \rightleftharpoons p + e^-$  and  $\bar{\nu}_e + p \rightleftharpoons n + e^+$ ) with the effective Lagrangian

coupling equal the weak coupling times  $\sin \theta_m$ .

We will consider two ranges of parameters, for which the  $\nu_e \rightarrow \nu_s$  oscillations occur on or off resonance. First, let us suppose that a resonant oscillation occurs somewhere in the core of the neutron star. Then the asymmetry in the neutrino emission comes from shift in the resonance point depending on the magnetic field<sup>2</sup>. Second, we will consider the off-resonance case, in which the asymmetry comes directly from the weak processes<sup>3</sup>.

### 3. Resonant Mikheev-Smirnov-Wolfenstein oscillations

Neutrino oscillations in a magnetized medium are described by an effective potential<sup>10</sup>

$$V(\nu_s) = 0 \quad (2)$$

$$V(\nu_e) = -V(\bar{\nu}_e) = V_0 (3 Y_e - 1 + 4 Y_{\nu_e}) \quad (3)$$

$$V(\nu_{\mu,\tau}) = -V(\bar{\nu}_{\mu,\tau}) = V_0 (Y_e - 1 + 2 Y_{\nu_e}) + \frac{eG_F}{\sqrt{2}} \left( \frac{3N_e}{\pi^4} \right)^{1/3} \frac{\vec{k} \cdot \vec{B}}{|\vec{k}|} \quad (4)$$

where  $Y_e$  ( $Y_{\nu_e}$ ) is the ratio of the number density of electrons (neutrinos) to that of neutrons,  $\vec{B}$  is the magnetic field,  $\vec{k}$  is the neutrino momentum,  $V_0 = 10 \text{ eV} (\rho/10^{14} \text{ g cm}^{-3})$ . The magnetic field dependent term in equation (4) arises from polarization of electrons and *not* from a neutrino magnetic moment, which in the Standard Model is small and which we will neglect. (A large neutrino magnetic moment can result in a pulsar kick through a somewhat different mechanism<sup>15</sup>.)

The condition for resonant MSW oscillation  $\nu_i \leftrightarrow \nu_j$  is

$$\frac{m_i^2}{2k} \cos 2\theta_{ij} + V(\nu_i) = \frac{m_j^2}{2k} \cos 2\theta_{ij} + V(\nu_j) \quad (5)$$

where  $\nu_{i,j}$  can be either a neutrino or an anti-neutrino.

In the presence of the magnetic field, condition (5) is satisfied at different distances  $r$  from the center, depending on the value of the  $(\vec{k} \cdot \vec{B})$  term in (5). The average momentum carried away by the neutrinos depends on the temperature of the region from which they escape. The deeper inside the star, the higher is the temperature during the neutrino cooling phase. Therefore, neutrinos coming out in different directions carry momenta which depend on the relative orientation of  $\vec{k}$  and  $\vec{B}$ . This causes the asymmetry in the momentum distribution.

The surface of the resonance points is

$$r(\phi) = r_0 + \delta \cos \phi, \quad (6)$$

where  $\cos \phi = (\vec{k} \cdot \vec{B})/k$  and  $\delta$  is determined by the equation  $(dN_n(r)/dr)\delta \approx e(3N_e/\pi^4)^{1/3} B$ . This yields<sup>2</sup>

$$\delta = \frac{e\mu_e}{\pi^2} B \sqrt{\frac{dN_n(r)}{dr}}, \quad (7)$$

where  $\mu_e \approx (3\pi^2 N_e)^{1/3}$  is the chemical potential of the degenerate (relativistic) electron gas.

Assuming a black-body radiation luminosity  $\propto T^4$ , the asymmetry in the momentum distribution is<sup>2</sup>

$$\frac{\Delta k}{k} = \frac{4e}{3\pi^2} \left( \frac{\mu_e}{T} \frac{dT}{dN_n} \right) B. \quad (8)$$

To calculate the derivative in (8), we use the relation between the density and the temperature of a non-relativistic Fermi gas. Finally,

$$\frac{\Delta k}{k} = \frac{4e\sqrt{2}}{\pi^2} \frac{\mu_e \mu_n^{1/2}}{m_n^{3/2} T^2} B = 0.01 \left( \frac{B}{3 \times 10^{15} \text{G}} \right) \quad (9)$$

if the neutrino oscillations take place in the core of the neutron star, at density of order  $10^{14} \text{ g cm}^{-3}$ . The neutrino oscillations take place at such a high density if one of the neutrinos has mass in the keV range, while the other one is much lighter. The magnetic field of the order of  $10^{15} - 10^{16} \text{ G}$  is quite possible inside a neutron star, where it is expected to be higher than on the surface. (In fact, some neutron stars, dubbed magnetars, appear to have surface magnetic fields of this magnitude.)

The region of parameters for which the asymmetric emission of sterile neutrinos would result in a sufficient pulsar kick is shown in Fig. 1. Obviously, the mass has to be in the keV range for the resonance to occur in the core of the neutron star. Theoretical models of neutrino masses can readily produce a sterile neutrino with a required mass<sup>16,17</sup>.

Some comments are in order. First, a similar kick mechanism, based entirely on active neutrino oscillations (and no steriles) could also work if the resonant oscillations took place between the electron and tau neutrinos<sup>1,18</sup>. This, however, would require the mass difference between two neutrinos to be of the order of 100 eV, which is ruled out. Second, the neutrino kick mechanism was criticized incorrectly by Janka and

Raffelt<sup>19</sup>. It was subsequently shown by several authors<sup>20,18</sup> that Janka and Raffelt made several mistakes, which is why their estimates were different from eq. (9). In particular, Janka and Raffelt neglected the neutrino absorptions outside the core, set the neutrino opacities to be equal to each other, regardless of the neutrino flavor (which is incorrect<sup>20</sup>), and neglected the change in the neutrino flux due to the  $1/r^2$  effect of the spherical outflow<sup>18</sup>.

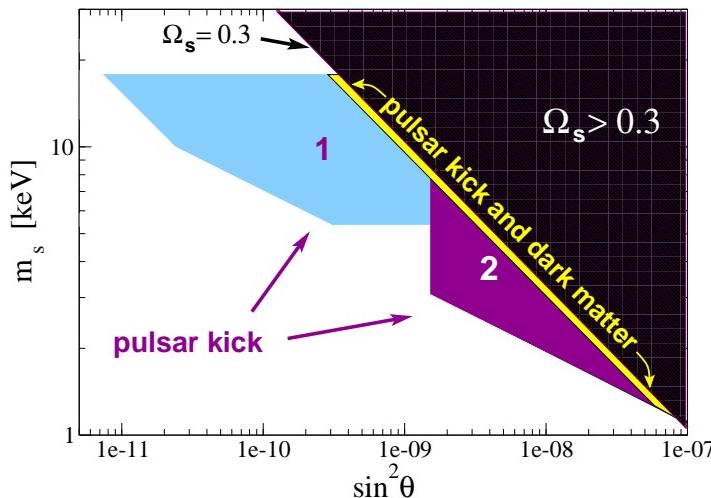


Figure 1. The range of parameters for the sterile neutrino mass and mixing. Regions 1 and 2 correspond to parameters consistent with the pulsar kicks for (1) resonant and (2) off-resonant transitions, respectively. Both regions overlap with a band in which the sterile neutrino is dark matter.

#### 4. Off-resonant oscillations

For somewhat lighter masses, the resonant condition is not satisfied anywhere in the core. In this case, however, the off-resonant production of sterile neutrinos in the core can occur through ordinary urca processes. A weak-eigenstate neutrino has a  $\sin^2 \theta$  admixture of a heavy mass eigenstate  $\nu_2$ . Hence, these heavy neutrinos can be produced in weak processes with a cross section suppressed by  $\sin^2 \theta$ .

Of course, the mixing angle in matter  $\theta_m$  is not the same as it is in vacuum, and initially  $\sin^2 \theta_m \ll \sin^2 \theta$ . However, as Abazajian, Fuller, and Patel<sup>12</sup> have pointed out, in the presence of sterile neutrinos the mixing angle in matter quickly evolves toward its vacuum value. When  $\sin^2 \theta_m \approx \sin^2 \theta$ , the production of sterile neutrinos is no longer suppressed, and they can take a fraction of energy out of a neutron star.

Following Abazajian, Fuller, and Patel<sup>12</sup>, we have estimated the time it takes for the matter potential to evolve to zero from its initial value  $V^{(0)}(\nu_e) \simeq (-0.2...+0.5)V_0$ . The time scale for this change to occur through neutrino oscillations off-resonance is

$$\begin{aligned} \tau_v^{\text{off-res}} &\simeq \frac{4\sqrt{2}\pi^2 m_n}{G_F^3 \rho} \frac{(V^{(0)}(\nu_e))^3}{(\Delta m^2)^2 \sin^2 2\theta} \frac{1}{\mu^3} \\ &\sim \frac{6 \times 10^{-9} s}{\sin^2 2\theta} \left( \frac{V^{(0)}(\nu_e)}{0.1 \text{eV}} \right)^3 \left( \frac{50 \text{MeV}}{\mu} \right)^3 \left( \frac{10 \text{keV}^2}{\Delta m^2} \right)^2. \end{aligned} \quad (10)$$

As long as this time is much smaller than 10 seconds, the mixing angle in matter approaches its value in vacuum in time for the sterile neutrinos to take out some fraction of energy from a cooling neutron star.

The urca processes produce ordinary neutrinos with some asymmetry depending on the magnetic field<sup>9</sup>. The same asymmetry is present in the production cross sections of sterile neutrinos. However, unlike the active neutrinos, sterile neutrinos escape from the star without rescattering. Therefore, the asymmetry in their emission is not washed out as it is in the case of the active neutrinos<sup>14</sup>. Instead, the asymmetry in emission is equal the asymmetry in production.

The number of neutrinos  $dN$  emitted into a solid angle  $d\Omega$  can be written as

$$\frac{dN}{d\Omega} = N_0(1 + \epsilon \cos \Theta_\nu), \quad (11)$$

where  $\Theta_\nu$  is the angle between the direction of the magnetic field and the neutrino momentum, and  $N_0$  is some normalization factor. The asymmetry parameter  $\epsilon$  is equal

$$\epsilon = \frac{g_V^2 - g_A^2}{g_V^2 + 3g_A^2} k_0 \left( \frac{E_s}{E_{\text{tot}}} \right), \quad (12)$$

where  $g_V$  and  $g_A$  are the axial and vector couplings,  $E_{\text{tot}}$  and  $E_s$  are the total neutrino luminosity and the luminosity in sterile neutrinos, respectively. The number of electrons in the lowest Landau level,  $k_0$ , depends on the magnetic field and the chemical potential  $\mu$  as shown in Fig. 2.

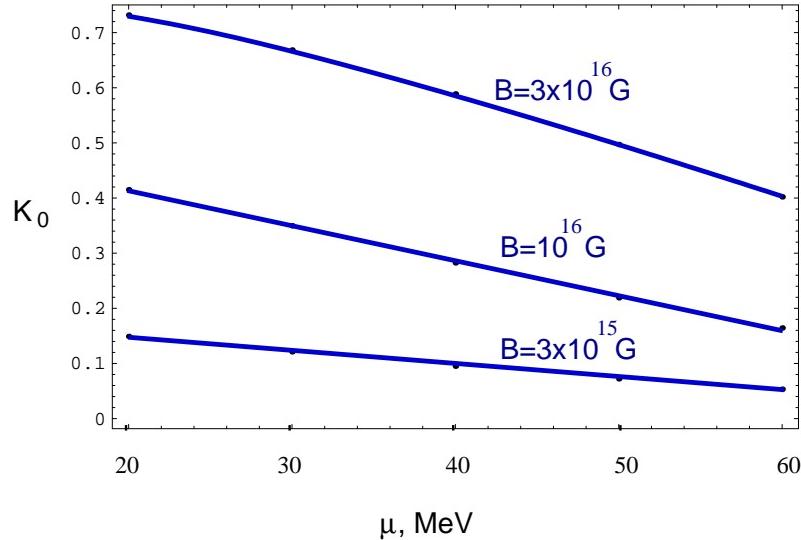


Figure 2. The fraction of electrons in the lowest Landau level as a function chemical potential. The value of the magnetic field in the core of a neutron star is shown next to each curve.

The momentum asymmetry in the neutrino emission is

$$\epsilon \sim 0.02 \left( \frac{k_0}{0.3} \right) \left( \frac{r_E}{0.5} \right), \quad (13)$$

where  $r_E$  is the fraction of energy carried by the sterile neutrinos. To satisfy the constraint based on the observation of neutrinos from supernova SN1987A, we require that  $r_E < 0.7$ . The asymmetry in equation (13) can be of the order of the requisite few per cent for magnetic fields  $10^{15} - 10^{16}$  G, as can be seen from Fig. 2.

Surface magnetic fields of pulsars are estimated to be of the order of  $10^{12} - 10^{13}$  G. However, the magnetic field inside a neutron star may be much higher<sup>21,22,23</sup>, probably up to  $10^{16}$  G. The existence of such a strong magnetic field is suggested by the dynamics of formation of the neutron stars, as well as by the stability of the poloidal magnetic field outside the pulsar<sup>22</sup>. Moreover, the discovery of soft gamma repeaters and their identification as magnetars<sup>23</sup>, *i.e.*, neutron stars with *surface* magnetic fields as large as  $10^{15}$  G, gives one a strong reason to believe that the interiors of many neutron stars may have magnetic fields as large as  $10^{15} - 10^{16}$  G and that only in some cases this large magnetic field breaks out to the surface. There are also plausible physical mechanisms capable of generating such a

large magnetic field inside a cooling neutron star<sup>21,22,24</sup>.

If the magnetic field inside a neutron star has a large non-dipole component, the neutrino kick is off-centered. Such a kick can probably explain the unusually fast rotation of pulsars<sup>11</sup>.

## 5. Sterile neutrinos as dark matter; observational consequences

Very few hints exist as to the nature of cosmological dark matter. We know that none of the Standard Model particles can be the dark matter, and we also know that the dark matter particles should either be weakly interacting or very heavy (or both). Theoretical models have provided plenty of candidates. For example, the supersymmetric extensions of the Standard Model predict the existence of a number of additional particles, which include two dark matter candidates: the lightest supersymmetric particle (LSP) and the SUSY Q-balls. These are plausible candidates, which were discussed in a number of talks at this conference. In the absence of observational hints, one naturally relies on theoretical models in planning experimental searches for dark matter.

The parameter space allowed for the pulsar kicks<sup>3</sup> overlaps nicely with that of dark-matter sterile neutrinos<sup>12,13</sup>. Sterile neutrinos in this range may soon be discovered<sup>25</sup>. Relic sterile neutrinos with mass in the 1-20 keV range can decay into a lighter neutrino and a photon. The X-ray photons should be detectable by the X-ray telescopes. Chandra and XMM-Newton can exclude part of the parameter space<sup>25</sup>. The future Constellation-X can probably explore the entire allowed range of parameters.

If inflation ended with a low-temperature reheating<sup>26</sup>, the allowed parameter space for the pulsar kicks extends to much lower masses and larger mixing angles<sup>26</sup>.

In the event of a nearby supernova, the neutrino kick can produce gravity waves that could be detected by LIGO and LISA<sup>27,28</sup>.

Active-to-sterile neutrino oscillations can give a neutron star a kick. However, if a black hole is born in a supernova, it would not receive a kick, unless it starts out as a neutron star and becomes a black hole later, because of accretion. (The latter may be what happened in SN1987A, which produced a burst of neutrinos, but no radio pulsar.) If the central engines of the gamma-ray bursts are compact stars, the kick mechanism acting selectively on neutron stars and not black holes could probably explain the short bursts as interrupted long bursts<sup>29</sup>.

Since one does not expect a significant correlation between the magnetic field inside a hot neutron star (while this field is, presumably, growing via the dynamo effect) and the eventual exterior field of a radio pulsar, the neutrino kick mechanism does not predict any  $B - v$  correlation.

To summarize, asymmetric neutrino emission from a cooling neutron star can explain the observed pulsar velocities. The necessary condition for this mechanism to work is the existence of a sterile neutrino with mass in the 1–20 keV range and a small mixing with ordinary neutrinos. It is intriguing that the same particle is a viable dark matter candidate. The nature of cosmological dark matter is still unknown. We know that at least one particle beyond the Standard Model must exist to account for dark matter. This particle may come as part of a “package”, for example, if supersymmetry is right. However, it may be that the dark matter particle is simply an  $SU(2) \times U(1)$  singlet fermion, which has a small mixing with neutrinos. Future observations of X-ray telescopes have the potential to discover the relic sterile neutrinos by detecting keV photons from the sterile neutrino decay in clusters of galaxies. If gravitational waves are detected from a nearby supernova, the signal may show the signs of a neutron star being accelerated by an asymmetric neutrino emission.

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